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RECONSTRUCTION OF PERIODIC BOUNDARY BETWEEN DIELECTRIC MEDIA

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Boundary shape analysis and reconstruction is important area of research [1-4], however there is a shortage of rigorous approaches. We propose a robust and clear modification of the known C method [5,6] for solving the wave scattering by arbitrarily shaped surface. This approach makes a reliable basis for solution of recognition problem: the reconstruction of surface profile and material parameters of media from known data on the scattered field.

For the ***direct problem*** solution, the C-method in combination with α regularisation has been chosen [5,6]. This enables us to reduce the original 2D problem of linearly polarized plane wave diffraction by an arbitrary boundary of dielectric media to operator equation

$$\alpha X + AX = C, \tag{1}$$

where α is regularizing parameter, A is a self-adjoint positively defined compact operator in corresponding Hilbert space $H = l_2 \times l_2$. The entries of unknown vector X are coefficients of scattered field that is expanded over eigenfunctions of C-method [5,6]. Vector C is defined by the excitation field. For boundary shape between media and given dielectric parameters the equation (1) can be solved efficiently by means of truncation method. The choice of regularising parameter α may be done on the base of generalised residual principle [7,8]. The input data for correspondent ***inverse problem*** are complex amplitudes $R = \left(R_n(\lambda)\right)_{n=-N}^{N}$ of reflected propagating waves, λ is a wavelength. We suppose that this data are known in certain wavelength range $[\lambda_1, \lambda_2]$. Besides the period of boundary shape and dielectric parameters of media are also known. It is necessary do find out by these input data the function, describing the boundary of two media. Let $a = \left(a_n\right)_{n=-\infty}^{\infty}$ be Fourier coefficients of this boundary function. The solution of operator equation (1) gives the mapping that associates set of $a = \left(a_n\right)_{n=-\infty}^{\infty}$ with set of complex amplitudes $R = \left(R_n(\lambda)\right)_{n=-N}^{N}$. Thus, on certain set of vectors $D_F \subset l_2$ the non linear operator

$$F(a,\lambda) = R(\lambda), \quad \lambda \in [\lambda_1, \lambda_2] \tag{2}$$

is defined. Having found out from (2) the Fourier coefficients $a = \left(a_n\right)_{n=-\infty}^{\infty}$, we can, by summation of Fourier series with approximate in l_2-space metric coefficients [7], derive the profile function. Formally, the scheme can be outlined as follows. Let $Y(\lambda)$, that is the solution of (2), is the set of operator F values. Introduce on $Y(\lambda)$ the norm according following

formula $\|R(\lambda)\|_1^2 = \sum_{-N}^{N} |R(\lambda)|^2 \frac{\cos\theta_n}{\cos\theta}$. Here following notations are used: θ_n are angles of diffracted field, θ is angle of incident field. Consider the functional that is given in definitional domain D_F of operator F

$$\Phi(a)=\sum_{m=1}^{M}\sum_{n=-P(\lambda_n)}^{P(\lambda_n)}\left|R_n^e(\lambda_m)-R_n^M(\lambda_m)\right|^2\frac{\rho_n(\lambda_m)}{\kappa\sqrt{\varepsilon_1\mu}}+\gamma\sum_{n=-Q}^{Q}\left|a_n\right|^2\left(1+n^{2R}\right) \qquad (3)$$

where $\gamma > 0$, is the parameter of regularization, $R > 0$, $\rho_n = \sqrt{\kappa_m^2\varepsilon_1\mu - n^2}$, $\kappa_m = d/\lambda_m$, d is a period of boundary, $\lambda_n \in [\lambda_1, \lambda_2]$, $a(v)=\sum_{n=-Q}^{Q}a_n e^{inv}$. Vector $a_\gamma^p = \left(a_{\gamma n}\right)_{n=-p}^{p}$, which provides the functional (3) with minimum we consider to be a solution to (2).

Numerical experiments. On the basis of the approach developed the numerical algorithms for the solving (1) and (2) have been implemented. The search of vector a_γ^p is organized by means of regularized quasi Newton's method with step adjustment, using only fist derivatives. The minimum residual method is applied for the choice of regularizing parameter γ. Based on (1), we simulated input data $R^e(\lambda_m)=\left(R_n^e(\lambda_m)\right)_{n=-N}^{N}$, $m=1,2...P$ for several boundaries between media characterized by profiles:

$$a_1(y)=h\left[0.5+\frac{\pi^3 y}{6d}\left(\frac{2y}{d}-1\right)\left(1-\frac{y}{d}\right)\right], \quad a_2(y)=h\left[0.375+0.25\sin\left(\frac{2\pi y}{d}\right)+0.125\cos\left(\frac{4\pi}{d}\right)\right],$$

$$a_2(y)=h\left[0.5-\frac{4}{\pi^2}\left(\cos\left(\frac{2\pi y}{d}\right)+\frac{1}{9}\cos\left(\frac{6\pi}{d}\right)+\frac{1}{25}\cos\left(\frac{10\pi}{d}\right)\right)\right]$$

periodically continued from interval $[0,d]$ onto interval $(-\infty,+\infty)$. Parameters d and h satisfy the restriction $2\pi h/d \leq 1$. The wavelength of incident plane E polarized wave was varying within the range $0.5 \leq d/\lambda \leq 3.5$. Permittivity of the first medium has been chosen as $\varepsilon_1 = 1$ and of the second one as $\varepsilon_2 = 2.25$. Permeability of both media is $\mu = 1$. Functions $a_1(y), a_2(y)$ are chosen for they belong to two essentially different classes. Namely, function $a_2(y)$ has a finite series of its Fourier coefficients. In the contrary, function $a_1(y)$ and $a_3(y)$ have an infinite Fourier series, which Fourier coefficients have algebraic type of decaying only. Results of numerical tests are presented in Fig. 1. Solid lines correspond to the exact functions $a_i(y), i = 1,2,3$ Lines depicted as crosses are the graphs of functions $a_1^R(y)$ and $a_2^R(y)$ that have been defined via input data $R^e(\lambda_m)=\left(R_n^e(\lambda_m)\right)_{n=-N}^{N}$ according to above described algorithm. As they almost coincide with graphic accuracy, the deviations $10h^{-1}\left|a_i(y)-a_i^R(y)\right|, i = 1,2,3$ are presented in the same figures as doted lines. It worth to emphasize that maximum absolute value of deviation essentially decreases with value of points P increases (we remind that P is a number of values of incident plane wave wavelengths, for which the input data, $m=1,2,... P$, have been calculated) that is in compliance with given level of noise in input data $R(\lambda_m)=\left(R_n(\lambda_m)\right)_{n=-N}^{N}$. Rather good approximation used for starting values of Fourier coefficients guarantees the shape reconstruction with accuracy $10^{-2}-10^{-3}$. These algorithms are efficient tools for the study of influence of input data errors on the accuracy of boundary shape reconstruction.
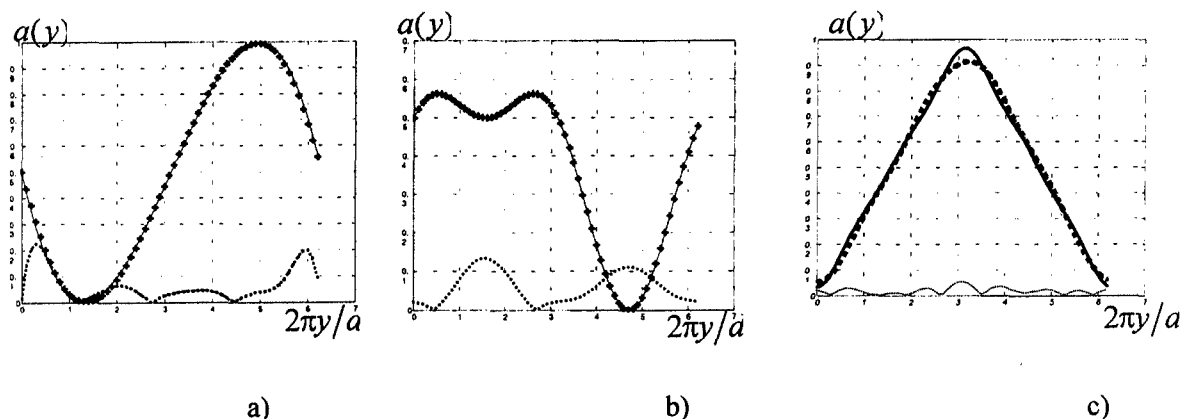
a) b) c)

Figure 1. Reconstruction of profiles given by functions $a_1(y)$ (a), $a_2(y)$ (b), and $a_3(y)$ (c) for the values $P = 6$, $\varepsilon_2 = 2.25$, $0.5 \le d/\lambda \le 3.5$, $2\pi h/d = 0.4$.

It is well known that one of the most complicated problems in solving unstable problems of reconstruction is the problem of matching regularizing parameter α with given level of errors in input data R_{mn}^e. We have demonstrated that such tradeoff can be obtained be means of residual method [7, 8]. We shall demonstrate this statement for the test problem for reconstruction shape of the surface, described by the function $a_2(y)$. Basing on the solution to direct problem we have calculated the input data $R_{mn}^e = R_m^e(\lambda_n)(1 + \gamma \cdot Rand)$ for various levels of relative error γ. The error $Rand$ has been simulated be the generator of random numbers with normal distribution. One of the numerical examples is presented in Fig. 2. In fragment a) you can see the characteristic behavior of relative error of profile reconstruction has been estimated according

formula $\quad \delta(\alpha) = \left(\dfrac{\sum_n |a_n^e - a_n^\alpha|^2 (1 + |n|^2)}{\sum_n |a_n^e|^2} \right)^{1/2}$, $\quad$ calculated $\quad$ for $\quad$ numbers $\quad$ of

$\alpha = 10^{-n}, n - 1, 2 \dots N;\ N \le 10$. Here a_n^e are exact values of function $a_2(y)$ Fourier coefficients. As it is clearly seen the function $\delta(\alpha)$ has pronounce minimum, that appears for all considered levels of error γ. In pictures b) and c) the results of boundary shape reconstruction re presented. Relying on our numerical experiment we can conclude that reconstruction can be performed with best accuracy for that value of α, that provides minimum to function $\delta(\alpha)$.

The residual method can provide reliable determination of optimal value of α according to

relation $\alpha_{opt} = \sup_\alpha (\alpha : \Delta_\alpha(\gamma) \le \gamma)$. Here $\quad \Delta_\alpha = \left(\dfrac{\sum_n |R_{mn}^e - R_{mn}^\alpha|^2}{\sum_n |R_{mn}^e|^2} \right)_\alpha^{1/2}$ is a relative residual of

input data R_{mn}^e and R_{mn}^α, that are the results of the solution of direct problem, calculated for function $a_2^\alpha(y)$ found out from minimization of (3) for given γ. From the results of numerical experiments we can see, that $\Delta_\alpha(\gamma)$ depends on α monotonically, and, thus α_{opt} is unique for each level of input data errors. The suggested algorithm, which performs reconstruction of shape of periodic boundary between two dielectric media relying on information about

diffraction harmonics that are known within certain interval of wavelength, requires certain starting approximation for Fourier coefficients of function $a(y)$ describing boundary. Such an approximation can be constructed by generalizing results of [10].
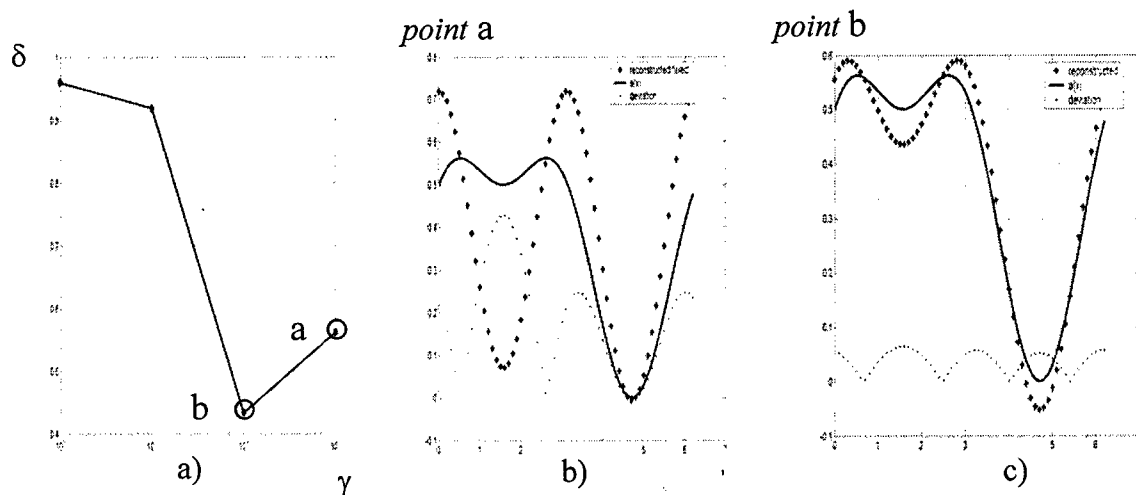
point a   *point* b

Fig.2

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